The *t*-test and ANOVA

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Background

- Question: Does a psychosocial inter-vention improve quality of life (QOL) following liver transplantation?
- Two groups of transplant patients
  - One gets treatment as usual (TAU)
  - Other gets psychosocial intervention (PSI)
- Measure QOL 6 months post-op

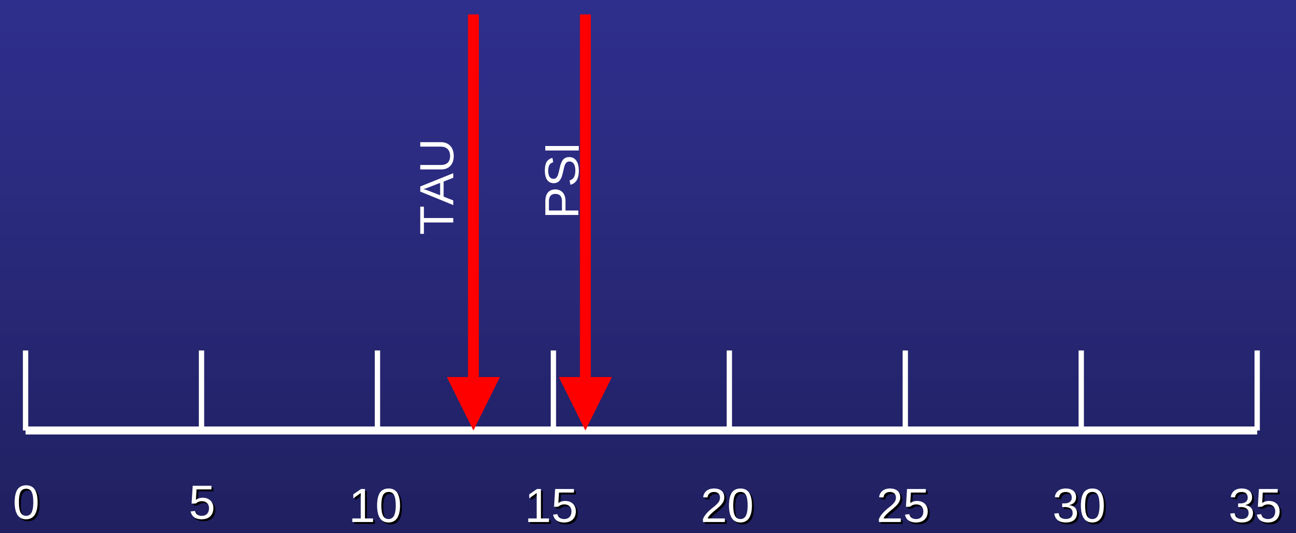
Background

- *A priori*, we say that a 10 point difference in QOL would be clinically important
- After the study, we find a difference between the groups
- Is the difference *statistically* significant?
- Answer: It all depends

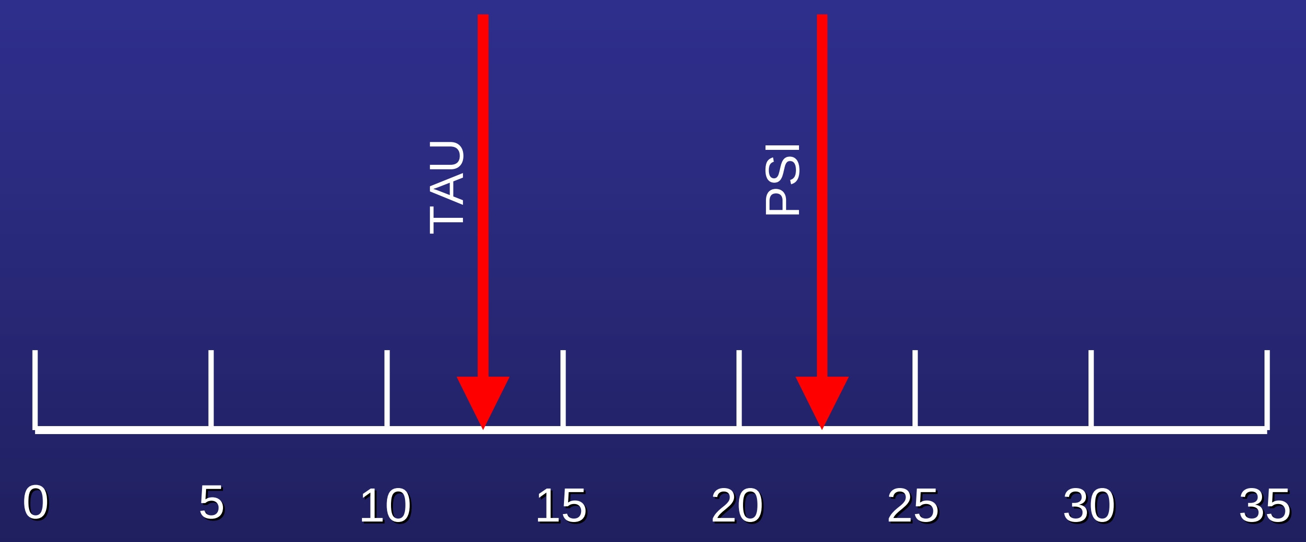
It Depends On …

1. How big is the difference?
   - The bigger the difference, the more likely the results are significant

Mean Difference

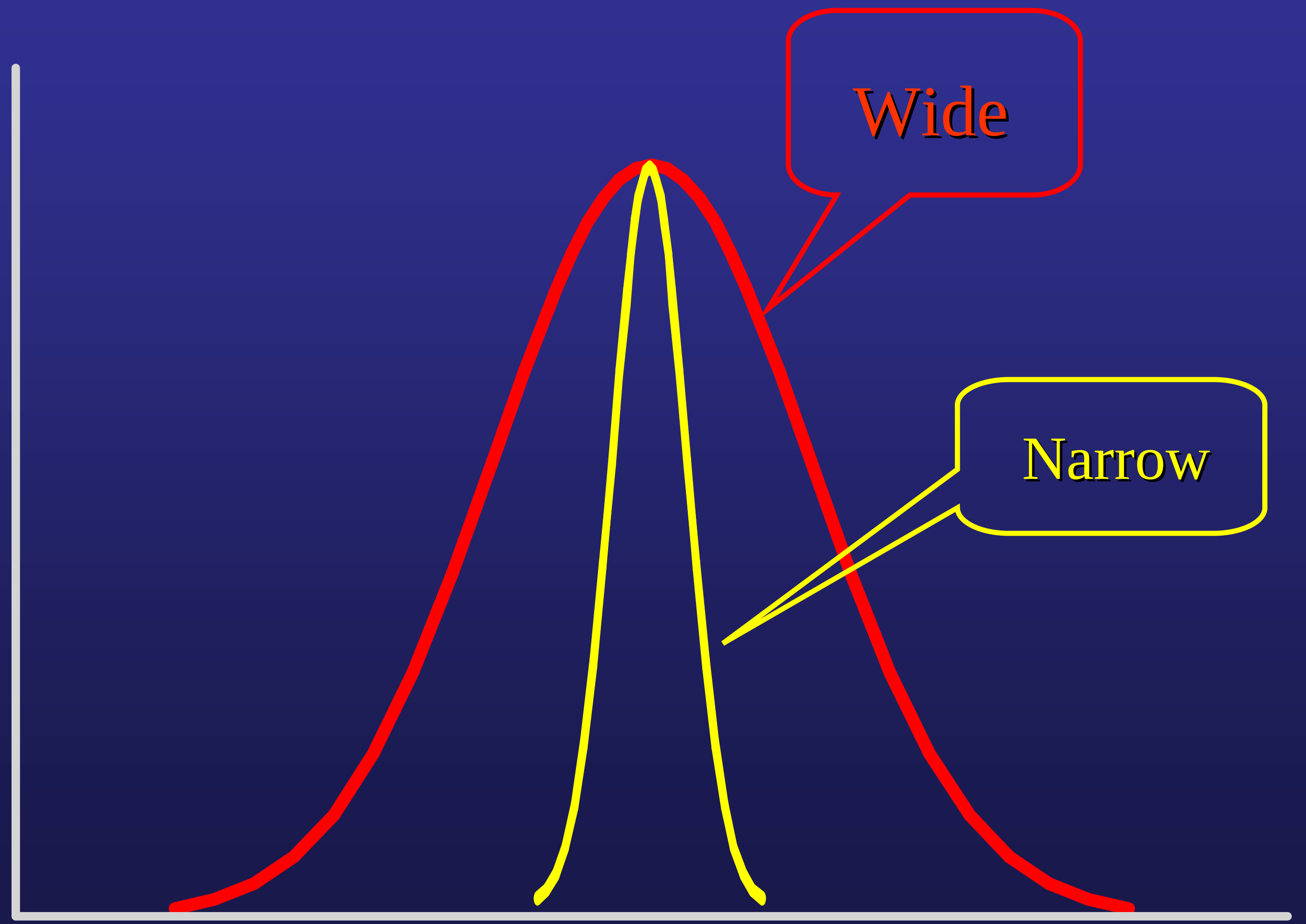


Mean Difference



- Does that mean that a large difference will be significant?
  - Again, it all depends
  - How accurately was each mean estimated?
  - If we repeated the study 100 times, how close to one another are the estimates of each group’s mean?

Spread of Mean Estimates



It Depends On …

- The spread of means is the *Standard Error of the Mean (SEM)*
- SEM is smaller if:
  - SD of the group is small
  - *N* is large
- The smaller the SEM, the more likely the results are significant

Summarizing

Difference between groups more likely to be significant if there is:

1. Large difference between means
2. Small SD in each group
3. Large *N* in each group

Writing it Out as an Equation

$$t = \frac{M_1 - M_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Conceptually

The *t*-test (and all statistical tests) are a ratio:

$$\frac{\text{Signal}}{\text{Noise}}$$

where *signal* is the size of effect;

noise is the amount of error

Adding More Groups

- Let's add a third group: occupational therapy (OT)
- How do we check for differences?
- Can do *t*-tests comparing:
  - TAU *versus* PSI
  - TAU *versus* OT
  - PSI *versus* OT

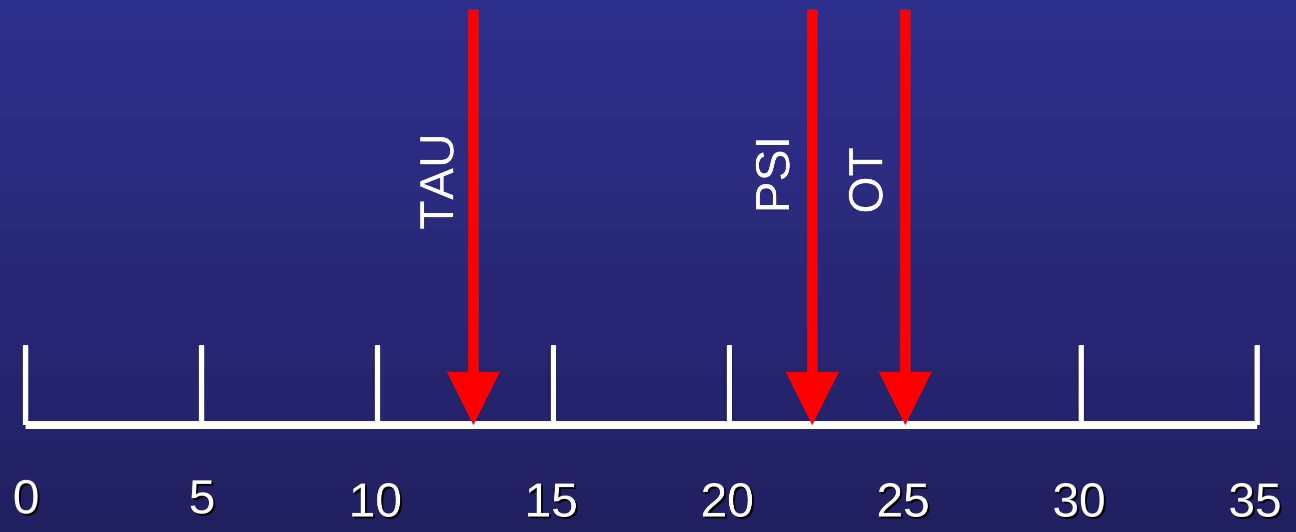
Problems with Many *t*-tests

- There are two problems with multiple *t*-tests:
  - Inflates probability of Type I error
  - Tests not independent of each other
    - If PSI > OT, and
    - OT > TAU, then
    - PSI *must be* > TAU

One-Way ANOVA

- Solution is one-way Analysis of Variance (ANOVA)
- Extension of *t*-test for >2 groups
- But – how do we calculate equivalent of ($M_1 - M_2$) when there are > 2 groups?

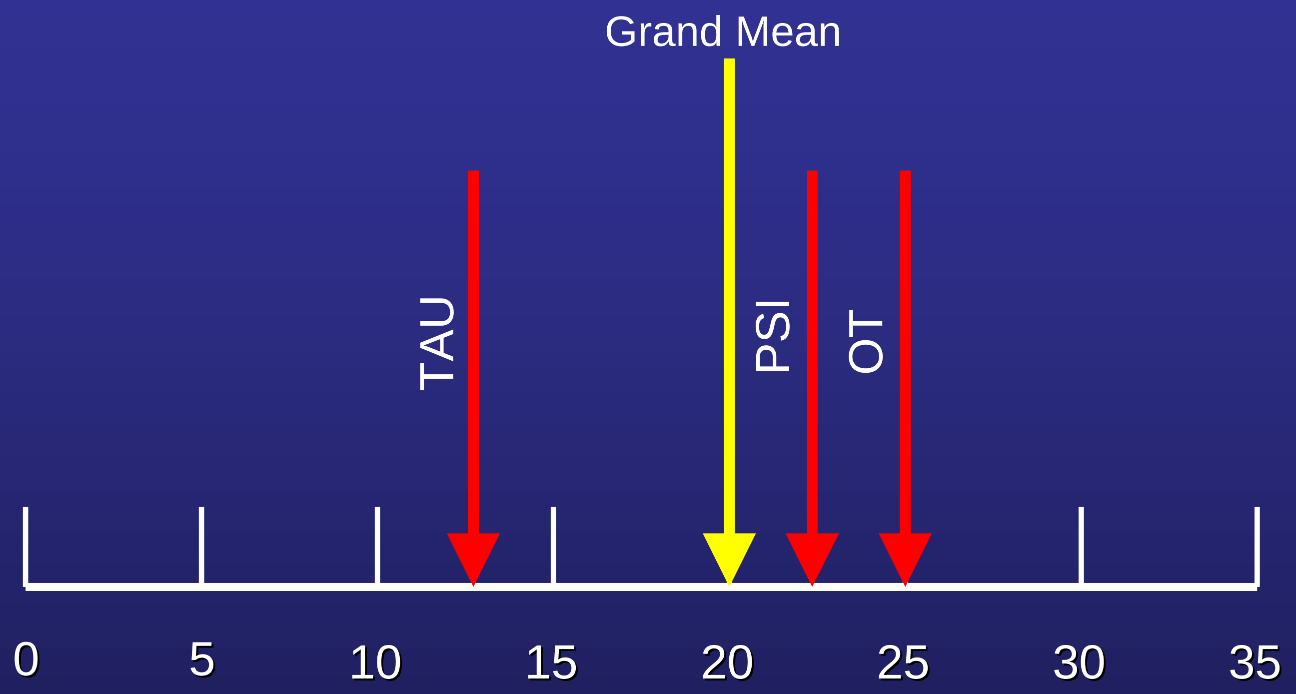
Difference for >2 Means



Difference for >2 Means

- Step 1: Find the Grand Mean (M_G) – the mean of the 3 means; or the mean of all people, irrespective of group
  - Assuming all groups have same sample size, it is (12. 5 + 22.5 + 25.0) / 3 = 60 / 3 = 20.0

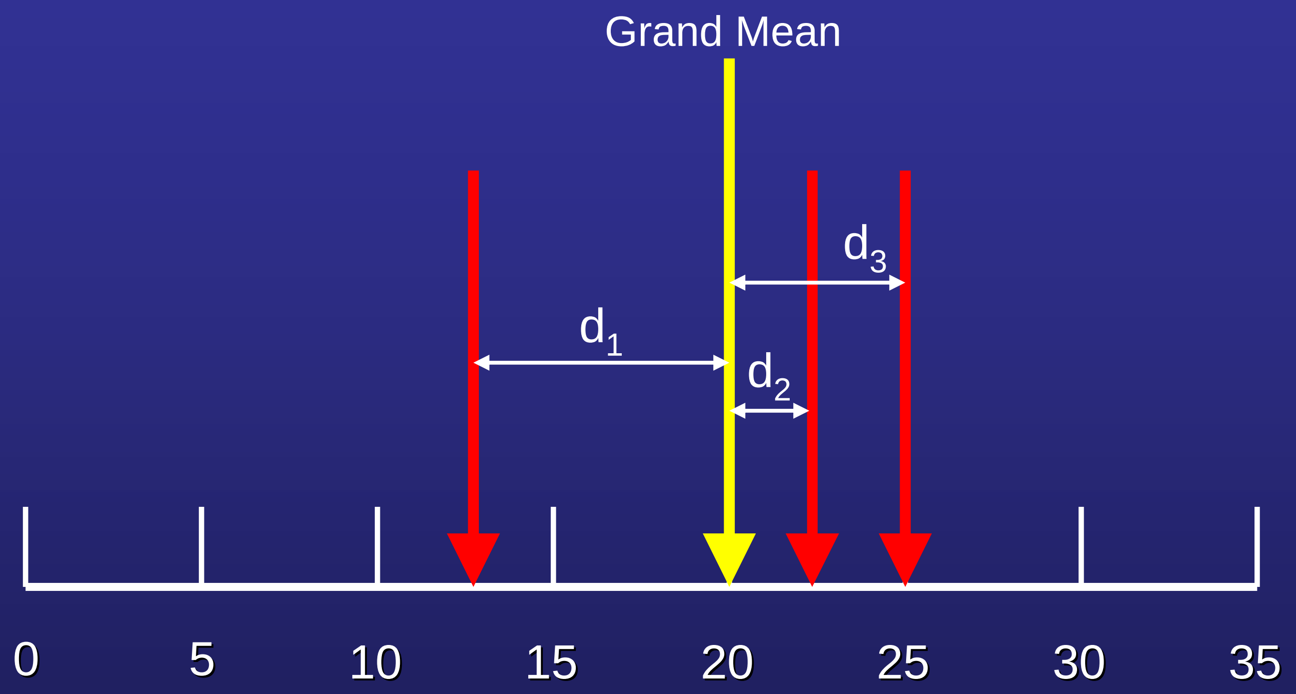
Difference for >2 Means



Difference for >2 Means

- Step 1: Find the Grand Mean (M_G)
- Step 2: Determine the deviation (*d*) of each mean from M_G

Difference for >2 Means



Difference for >2 Means

- Step 1: Find the Grand Mean (M_G)
- Step 2: Determine the deviation (*d*) of each mean from M_G
  - $d_1 = (12.5 - 20.0) = -7.5$
  - $d_2 = (22.5 - 20.0) = 2.5$
  - $d_3 = (25.0 - 20.0) = 5.0$

Difference for >2 Means

- We can't just add them up
- The answer will *always* be 0
  - E.g., (-7.5) + (2.5) + (5.0) = 0
- Square each value first
- Divide by number of means

Difference for >2 Means

- Step 1: Find the Grand Mean (M_G)
- Step 2: Determine the deviation (*d*) of each mean from M_G
- Step 3: Determine mean (average) squared deviation

Difference for >2 Means

- This is the numerator
- It is a *variance*; in this case, the variance *between* the groups
- The more the means are spread out, the larger the between-groups variance (i.e., the *signal*)

Difference for >2 Means

- What's the denominator (the *noise*)?
  - As with the *t*-test, it is the variance *within* each of the groups
- So, ANOVA is again signal-to-noise ratio
- For various reasons, we call the variance the *mean square*

One-Way ANOVA

- So, the ANOVA is:

$$F = \frac{\text{Mean Square}_{\text{Between Groups}}}{\text{Mean Square}_{\text{Within Groups}}}$$

Conceptually

- There is *variance* (variability) among all of the outcomes
- ANOVA tries to find the sources of this variance:
  - Due to differences between group
  - Variability within the groups (error)
  - "Error" better termed "unexplained variance"

Beyond One-Way ANOVA

- Can look for other factors to account for (explain) error variance; e.g., sex
- Design becomes more complicated

Factorial ANOVA

	TAU	PSI	OT
Males			
Females			

Factorial ANOVA

- Can add more factors
- Limited by sample size, ability to understand results
- Adding factors that don't explain variance make the results *worse*
- Best guide is *judgement*