Categorical data

Also called

- discrete data
- frequency data
- qualitative data
- data on nominal or ordinal scale

as opposed to

- quantitative data
- numerical data
- continous data
- data on interval or ratio scale

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Categories are coded: codes can even be letters or text.

Don't calculate statistics such as average, median, SD, etc. from the codes, even if they are numbers!

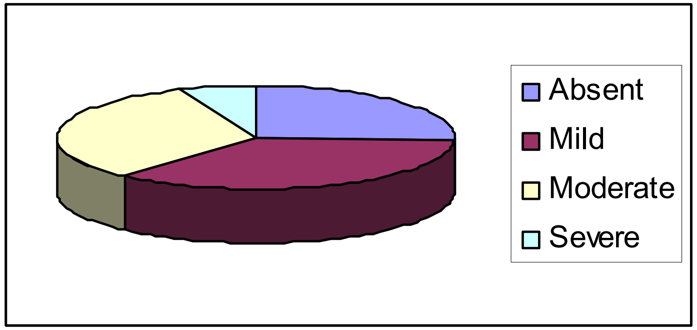
Analysis of a single categorical variable

- Frequency table
- Mode (=the most likely category)
- Barchart
- Pie chart

Severity of symptoms in a sample of 220 patients:

Severity	Absent	Mild	Moderate	Severe
Frequency	56	79	71	14
%	25.5	35.9	32.3	6.4

Mode (or modal category)



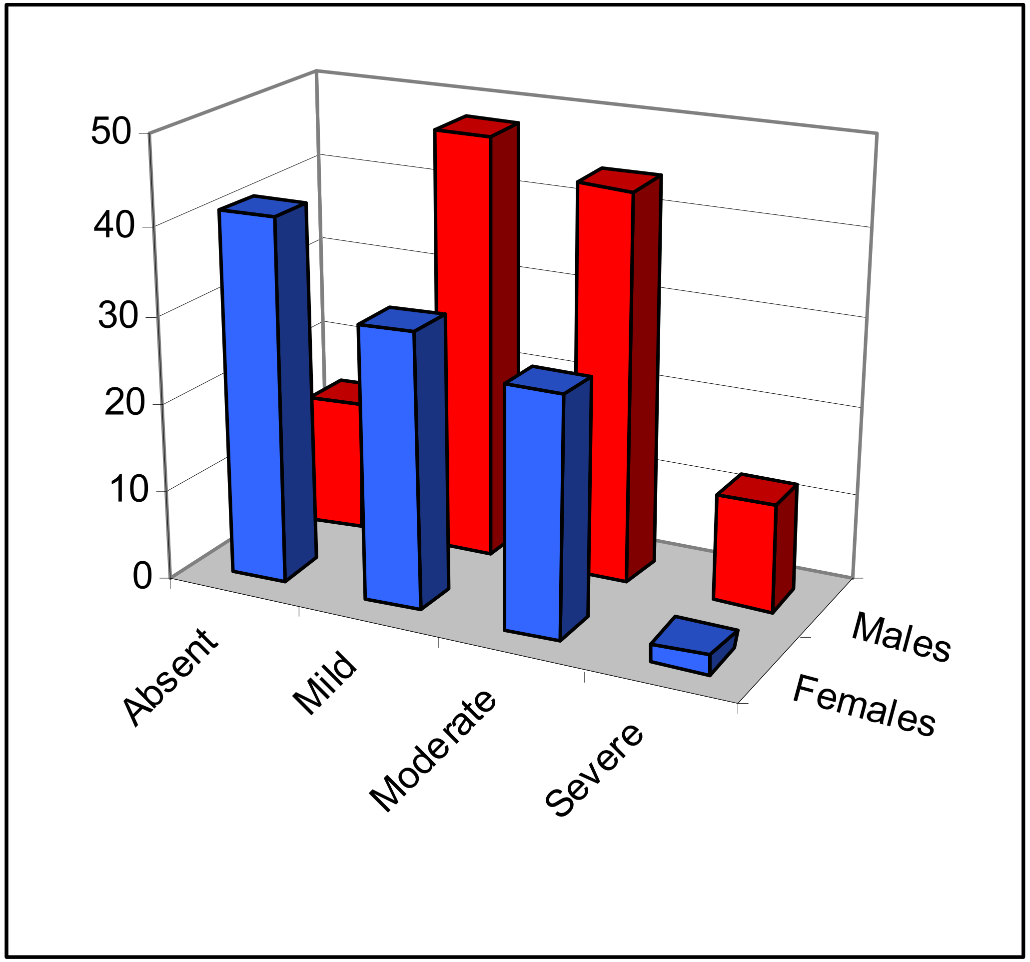
Analysis of the relationship between two categorical variables:

- Contingency table (=two-dimensional frequency table)
- Three-dimensional barchart
- Association measures
- Tests of independence

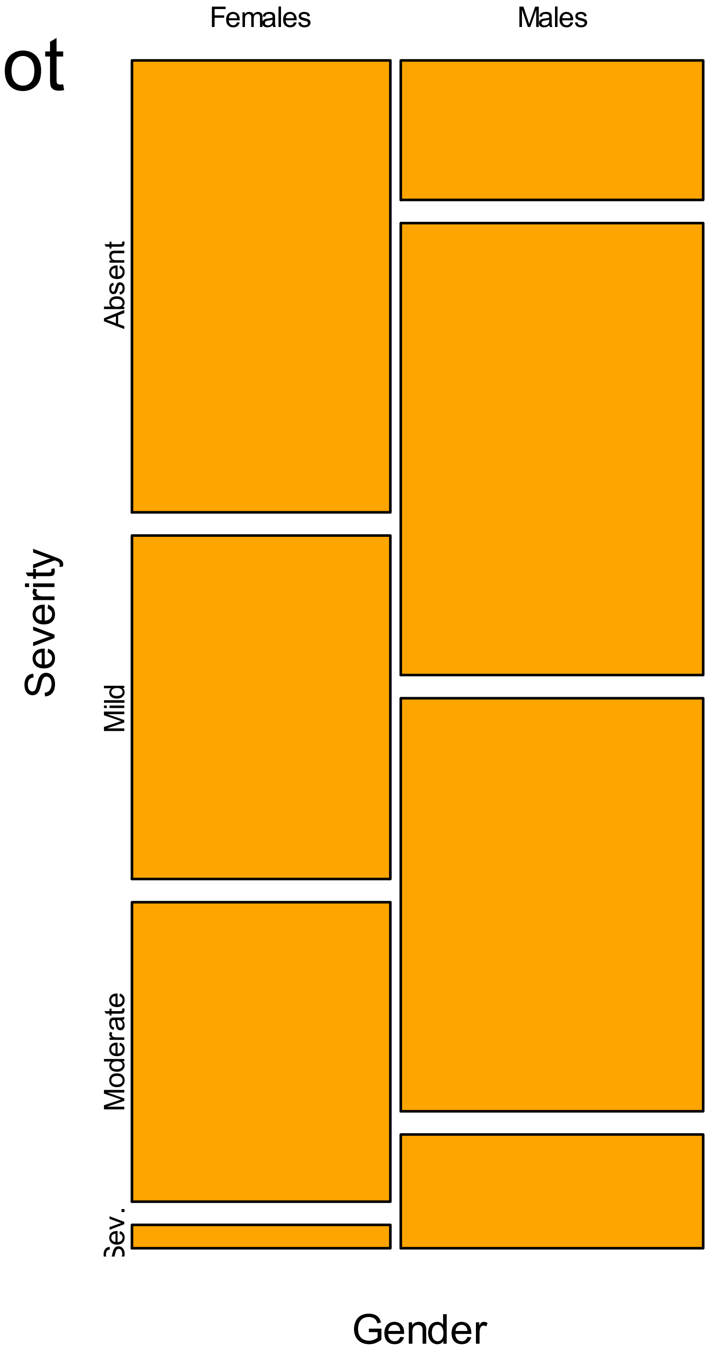
Severity of symptoms by gender:

	Absent	Mild	Moderate	Severe	Total
Females	41	31	27	2	101
Males	15	48	44	12	119
Total	56	79	71	14	220

3-D barchart



Mosaic plot



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I don't mean causally!

Please don't call it correlation!!!

Measures of association

…quantify how strong an association exists between *X* and *Y*.

The traditional setting (there are other variants as well):

0 — no association — *X* and *Y* are independent

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Most frequently used measures of association:

- Cramer's V
- Goodman and Kruskal's lambda

Good measures of association are invariant to changing the codes and/or the order of categories!

Correlation

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- Positive correlation: the more (better, higher, etc.) the *X*, the more (better, higher, etc.) the *Y*.
- Negative correlation: the more (better, higher, etc.) the *X*, the less (worse, lower, etc.) the *Y*.

Correlation coefficients

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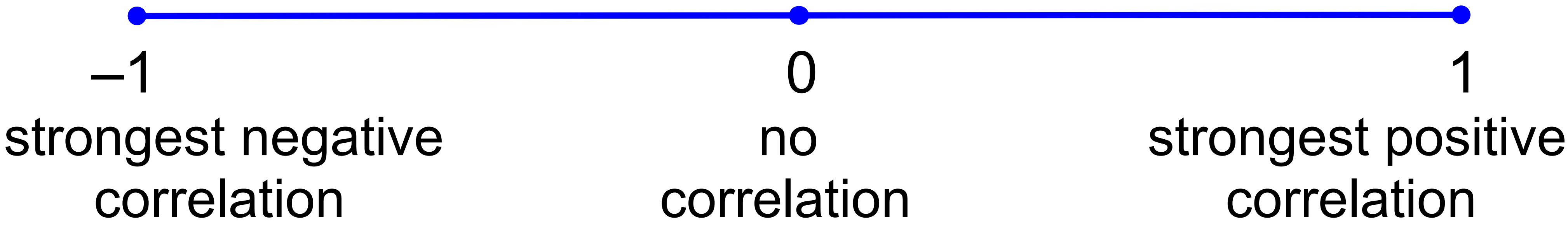

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strongest negative correlation	no correlation	strongest positive correlation
Ordering of the subjects according to their *X* values is exactly the inverse of their ordering according to their *Y* values	Independence of *X* and *Y* implies zero correlation (but zero correlation does not imply independence)	Ordering of the subjects according to their *X* values is fully identical with their ordering according to their *Y* values

Correlation coefficients applicable to categorical data:

- Kendall's tau
- Spearman's rho

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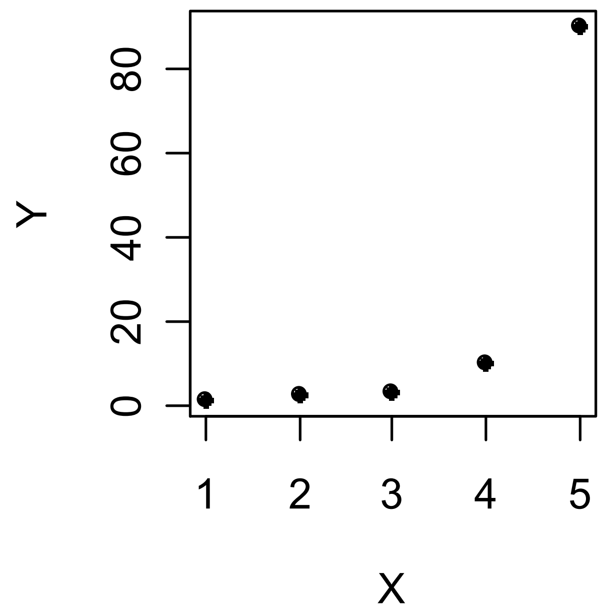
Example:

Let us have the data *X*: 1, 2, 3, 4, 5, and *Y*: 1, 2, 3, 10, 90, exhibiting a perfect monotonic relationship, so we expect a correlation of 1 between *X* and *Y*.

However, Pearson's coefficient gives just 0.76 while both Kendall's and Spearman's coefficients result in the right value 1.



Tests of independence

H_0: X and Y are independent

H_1: X and Y are not independent

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Available tests:

- Chi-squared test (also called Pearson's chi-squared test)
- Fisher's exact test

Small p-values ($p \leq 0.05$) indicate non-independence, i.e. presence of some kind of association between X and Y.

Be aware that the chi-squared test is valid only for large samples!

For small samples use Fisher's exact test instead!

Example:

Let us look at the contingency table gender by severity of symptoms, and test whether severity of symptoms is independent of gender!

Let us carry out both tests by the statistical software R!

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A free but absolute professional software with a full spectrum of the best analysis methods!

Joint analysis of several categorical variables

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Methods for the **joint analysis of several categorical and continuous variables** (again just some names…)

- Analysis of variance, general linear models
- Logistic regression, generalized linear models
- Discriminant analysis

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Also called **non-parametric methods**, as opposed to parametric methods.

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Parametric methods:

- Student's t-test are valid only for ***normally distributed*** data
- Pearson's correlation coefficient is valid only for ***normally distributed data***
- ANOVA is valid only if data follow the ***normal distribution in each group***

Nonparametric methods:

- Wilcoxon's signed rank test is valid for any ***continuous and symmetric*** distribution
- Sign test is valid for any ***continuous*** distribution
- Spearman's rank correlation is valid for any data on ***ordinal and interval scale***

Most frequently used distribution-free methods:

- Sign test (one sample, paired samples)
- Mood’s median test (two or more samples)
- Wilcoxon signed rank test* (one sample, paired samples)
- Wilcoxon rank sum test*
  also called Mann-Whitney U-test (two samples)
- Kruskal-Wallis test* (several samples)
- Confidence interval for the median (one sample)
- Spearman’s rank correlation coefficient*
- Kendall’s tau (correlation coefficient)

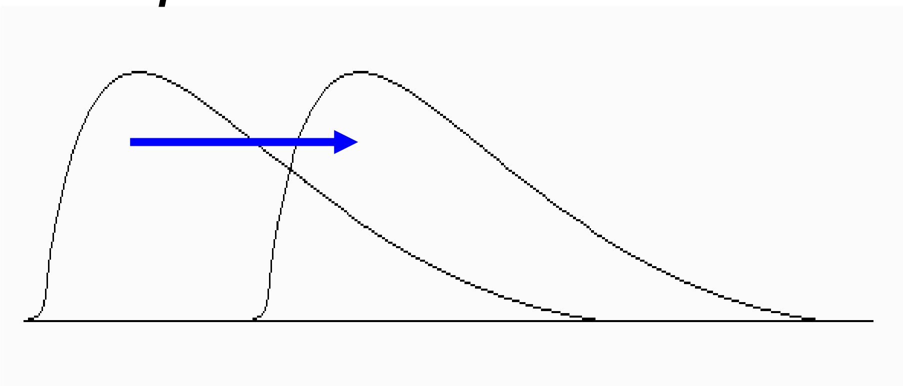
*rank-based methods

This is just a selection!

Be aware that **Wilcoxon rank sum test and Kruskal-Wallis test** in their original form are valid only if the variables to compare have **distributions of the same shape**!

Having the same shape means that the difference between the groups is simply a ***shift*** (which is in most cases irrealistic; then even the variances must be equal).

shapes are same here



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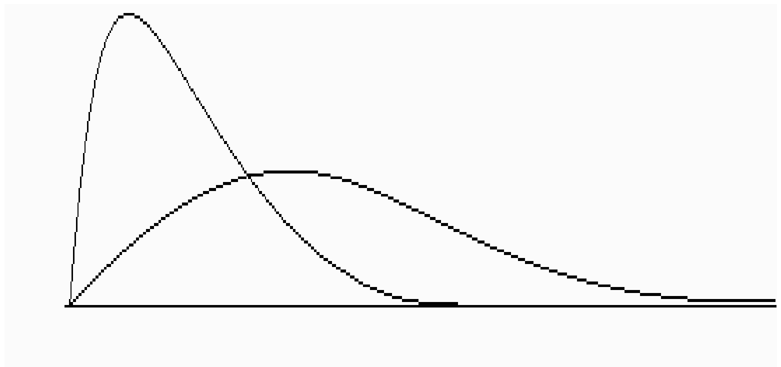

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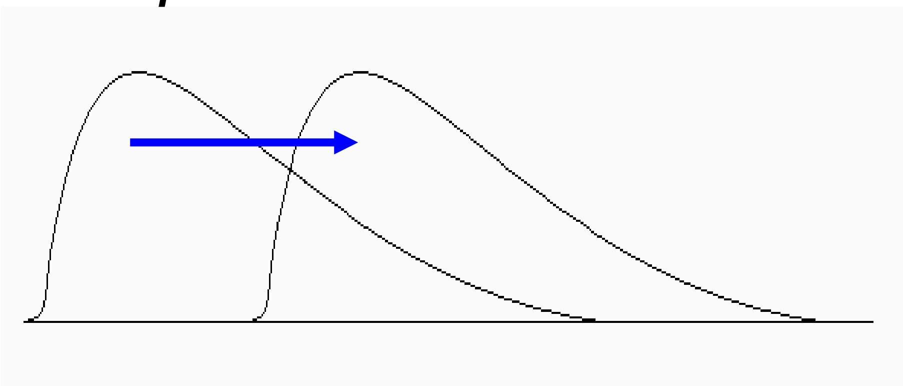

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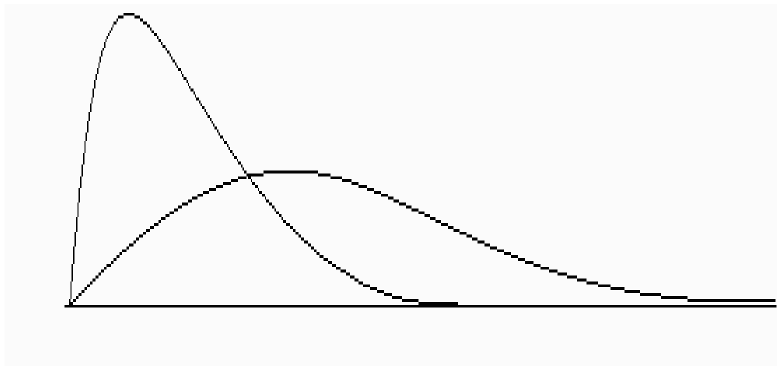

Fortunately, there are newer versions of these tests, which don't require this rather restrictive assumption! Check the latest literature!